Task 1 (40 points)
Show the identity (notation as in the lecture)

$$\int_{\partial A} \vec{F}(x,y) \cdot \mathrm{d}\vec{\ell}_\perp = \int_A \vec{\nabla} \cdot \vec{F}(x,y) \, \mathrm{d}A \,, \tag{1}$$

with reference to a small, but not necessarily infinitesimally small, reference area, namely, a rectangle with lower-left corner (x, y) and upper-right corner $(x + \delta x, y + \delta y)$.

Task 2 (40 points)
Show the identity (notation as in the lecture)

$$\oint_{\partial A} \vec{F}(x,y) \cdot \mathrm{d}\vec{\ell} = \int_A \left[\vec{\nabla} \times \vec{F} \right]_z \mathrm{d}A \tag{2}$$

with reference to a small, but not necessarily infinitesimally small, reference area, namely, a rectangle with lower-left corner (x, y) and upper-right corner $(x + \delta x, y + \delta y)$.

Does the contour ∂A encircle the area in the counterclockwise, or clockwise, direction?

Task 3 (40 points)
Calculate the complex closed-contour integral

$$I = \oint_C z^{-1} \, \mathrm{d}z = \oint_C \frac{1}{z} \, \mathrm{d}z \,, \tag{3}$$

where C is the edge of a rectangle of side length 2, encircled counter-clockwise. So, we would have $C = z_1 \to z_2 \to z_3 \to z_4 \to z_1$, with $z_1 = 1 + \mathrm{i}$, $z_2 = -1 + \mathrm{i}$, $z_3 = -1 - \mathrm{i}$, and $z_4 = 1 - \mathrm{i}$. *Calculate the integral by explicitly evaluating all individual line integrals separately, as stated in the task. Just quoting or using a general result will result in zero credit.*

The tasks are due Thursday, 04–FEB–2021.