Task 1 (unmarked)
Consider the following passage in the lecture notes,

$$\begin{aligned}\oint f(z)\mathrm{d}z &= \oint (f_1 + \mathrm{i} f_2)(\mathrm{d}x + \mathrm{i}\mathrm{d}y) \\ &= \oint (f_1 \mathrm{d}x - f_2 \mathrm{d}y) + \mathrm{i} \oint (f_2 \mathrm{d}x + f_1 \mathrm{d}y) \\ &= \int_{\partial A} \vec{F}^* \cdot \mathrm{d}\vec{\ell} - \mathrm{i} \int_{\partial A} [\, \vec{F}^* \times \hat{e}_z \,]_z \cdot \mathrm{d}\vec{\ell} \\ &= \int_{\partial A} (\vec{\nabla} \times \vec{F}^*)_z \mathrm{d}A - \mathrm{i} \int_{\partial A} [\, \vec{\nabla} \times (\vec{F}^* \times \hat{e}_z) \,]_z \mathrm{d}A \\ &= \int_A (\vec{\nabla} \times \vec{F}^*)_z \mathrm{d}A + \mathrm{i} \int_A \vec{\nabla} \cdot \vec{F}^* \mathrm{d}A \,. \end{aligned} \tag{1}$$

Summarize, once more, the connection of f and $\vec{F}^*$, as in the lecture notes. Then, write an essay for yourself in which you explain all intermediate steps. Optionally, consider an alternative formulation based on the identity

$$\mathrm{d}\vec{\ell}_\perp = \mathrm{d}\vec{\ell} \times \hat{e}_z \,. \tag{2}$$

Task 2 (unmarked)
Why is the function

$$f(z) = \mathrm{Re}(z) \tag{3}$$

not fulfilling the Cauchy–Riemann differential equations? Maybe, does it still fulfill (almost trivially) one of the two Cauchy–Riemann differential equations, and why? Which consequences result from your considerations? Is the imaginary part of a complex contour integral over $f(z)$ still path-independent, and if yes, why? *This question has a certain wit and is almost trivial, but one may need to think about it.*

The tasks are due Thursday, 18–FEB–2021.