Task 1 (40 points). Consider the entire derivation of the general expression for the divergence of a vector field, in general coordinates. Show that

$$\nabla_i \, A^i = \partial_i \, A^i + \frac{A^i}{\sqrt{\det g}} \, \partial_i \, \sqrt{\det g} = \frac{1}{\sqrt{\det g}} \, \partial_i \, \left(\sqrt{\det g} \, A^i \right) \, . \tag{1}$$

You are encouraged to use lecture notes, but it must be visile that you redid every step yourself. Include comments in your derivation!

Task 2 (40 points). Show that, in orthogonal curvilinear coordinates, one has

$$A_{(i)} = \frac{A^i}{h_{(i)}} \, , \qquad \nabla_i \, A^i = \frac{1}{h_{(1)} \, h_{(2)} \, h_{(3)}} \partial_i \, \left(h_{(1)} \, h_{(2)} \, h_{(3)} \, A^i \right) = \frac{1}{h_{(1)} \, h_{(2)} \, h_{(3)}} \partial_i \, \left(\frac{h_{(1)} \, h_{(2)} \, h_{(3)}}{h_{(i)}} \, A_{(i)} \right) \, . \tag{2}$$

Here, the $A_{(i)}$ are the physical components, while the A^i are the covariant components. The $h_{(i)}$ are the structure functions of the metric.

Task 3 (40 points). Simplify the expressions for the divergence of the $\vec{A}$ vector field, namely, $\nabla_i \, A^i$, as far as possible, for spherical and cylindrical coordinates. Compare your result with the literature!

The tasks are due on Thursday, 29–APR–2021.