

Task 1 (50 points)

Derive the Taylor expansion

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \mathcal{O}(x^4). \quad (1)$$

Explain the meaning of the term $\mathcal{O}(x^4)$. Explain the connection of the Taylor expansion to the Einstein formula

$$E = \sqrt{\vec{p}^2 c^2 + m^2 c^4} \quad (2)$$

with symbols explained as in the lecture. Where does the nonrelativistic kinetic energy $\vec{p}^2/(2m)$ appear in the formulas? **Make a plot, using a computer algebra system of your choice, of the approximations**

$$\sqrt{1+x} \approx 1 + \frac{x}{2}, \quad (3)$$

$$\sqrt{1+x} \approx 1 + \frac{x}{2} - \frac{x^2}{8}, \quad (4)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}, \quad (5)$$

and show, visually, how the approximations become better with increasing order.

Task 2 (50 points)

Download the L^AT_EX typesetting system, in a version appropriate for your favorite operating system. For Windows, you might download MikTeX. Compile the following example text (you may use the pdf version of the exercise, to be posted on the web page of the course, to copy and paste, or manually copy from the source code below):

```
\documentclass{article}
\begin{document}
Binomial formula:
\begin{equation}
(a + b)^2 = a^2 + 2 a b + b^2
\end{equation}
Pythagoras's theorem:
\begin{equation}
a^2 + b^2 = c^2
\end{equation}
\end{document}
```

Look up the Maxwell equations on the internet and write a L^AT_EX document with typeset Maxwell equations! You may use the integral form, or the differential form, or, even better, both!

The tasks are due Thursday, 02-SEP-2021. Have fun doing the problems!