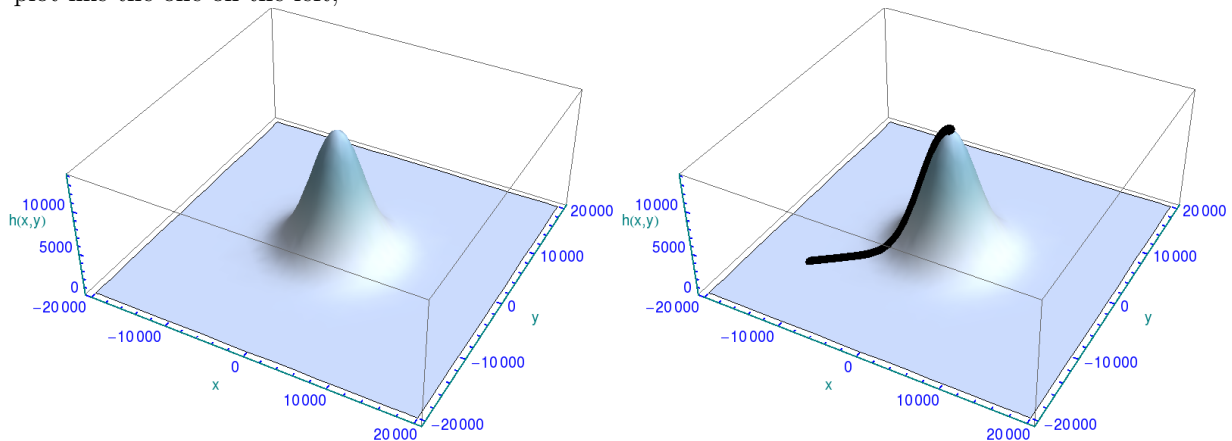


PLEASE READ THE TASKS CAREFULLY!!! **Task 1** (100 points)

Consider a mountain surface given by the equation

$$h(x, y) = A \exp[-B(x^2 + y^2)], \quad A = 14200 \text{ ft}, \quad B = \frac{1}{2 \times 10^7} \frac{1}{(\text{ft}^2)}. \quad (1)$$

Here, ft is the unit of foot. **Plot  $h(x, y)$  over a meaningful range of  $x$  and  $y$  values.** You might obtain a plot like the one on the left,



but you should label the  $x$  and  $y$ , and  $z$  axes with appropriate physical units (which can be given in feet or meters). Then, consider the path

$$\vec{s}(t) = \hat{e}_x s_x(t) + \hat{e}_y s_y(t), \quad s_x(t) = s_{0x} + v_x t, \quad s_y(t) = s_{0y} + v_y t, \quad (2)$$

$$s_{0x} = s_x(t=0) = -13000 \text{ ft}, \quad s_{0y} = s_y(t=0) = -10000 \text{ ft}, \quad v_x = \frac{13 \text{ ft}}{16 \text{ s}}, \quad v_y = \frac{5 \text{ ft}}{8 \text{ s}},$$

where  $s$  stands for the unit of second. **Plot  $h(\vec{s}(t))$  over a the range  $t \in (0, 16000 \text{ s})$  and overlay the plot with the one obtained above.** You might obtain a plot like the one on the right, above, but you should label the  $x$  and  $y$ , and  $z$  axes with appropriate physical units (which can be given in feet or meters).

**Show, by an explicit and complete analytic evaluation in terms of the general parameters  $A$ ,  $B$ ,  $s_{0x}$  and  $s_{0y}$ , that**

$$\frac{dh(\vec{s}(t))}{dt} = \vec{\nabla}h(\vec{r}) \Big|_{\vec{r}=\vec{s}(t)} \cdot \frac{d\vec{s}(t)}{dt} \quad (3)$$

**Show every step and every intermediate result for every single quantity on the left and right-hand sides of the equation!** Calculate the integral (numerically!)

$$\int_{t=0}^{t=16000 \text{ s}} dt \frac{dh(\vec{s}(t))}{dt} \approx 14200 \text{ ft} \quad (4)$$

and interpret your result geometrically, looking at the plot. **How fast do you reach the summit for the parameters  $v_x = 6.5 \text{ ft/s}$  and  $v_y = 5 \text{ ft/s}$ , for an identical start point of your way up the mountain?**

PLEASE READ THE TASKS CAREFULLY!!! **Task 2** (40+10 extra points)

(40 extra points) Show that the Green function of the two-dimensional Poisson equation is

$$G(x, y) = \frac{1}{2\pi} \ln \left( \frac{\sqrt{x^2 + y^2}}{a} \right) \quad \vec{\nabla}^2 G(x, y) = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) G(x, y) = 2\pi \delta^{(2)}(\vec{r}). \quad (5)$$

(10 extra points) If possible, relate your result to Gauss's and Stokes's theorem, and Cauchy's theorem, in two dimensions, as done in the lecture.

The tasks are due Thursday, 09-SEP-2021. Have fun doing the problems!