Task 1 (20 extra points)
Show that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\ln\left(\frac{\sqrt{x^2+y^2}}{a}\right)=0\,, \tag{1}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\frac{1}{\sqrt{x^2+y^2+z^2}}=0\,, \tag{2}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial \xi^2}\right)\frac{1}{x^2+y^2+z^2+\xi^2}=0\,, \tag{3}$$

provided

$$(x,y)\neq(0,0)\,,\qquad (x,y,z)\neq(0,0,0)\,,\qquad (x,y,z,\xi)\neq(0,0,0,0)\,. \tag{4}$$

How is the derived result related to the Green functions of the Poisson equation in two, three and four dimensions?

(Hint: For the first and the second of the above three equations, you may use your lecture notes and an appropriate notation of your choice, e.g., $||\vec{r}|| = \sqrt{x^2+y^2+z^2}$.)

The tasks are due Tuesday, 21-SEP-2021. Have fun doing the problems!