Task 1 (20 points)
Discuss the concepts of the Poisson equation, the Green function $g(\vec{r} - \vec{r}')$, the electrostatic potential $\Phi(\vec{r})$, and the charge distribution $\rho(\vec{r})$, in a short essay. Use complete English sentences. Your essay should contain the equation

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \mathrm{d}^3 r' \, \frac{1}{|\vec{r} - \vec{r}'|} \, \rho(\vec{r}') \tag{1}$$

Task 2 (20 points)
Calculate $\Phi(\vec{r})$ for the charge distribution

$$\rho(\vec{r}') = q_0 \, \delta^{(3)}(\vec{r}' - \vec{r}_0) \,. \tag{2}$$

Task 3 (60 points+20 extra points)
Based on the integral representation

$$\Gamma(z) = \int_0^\infty \mathrm{d}t \, t^{z-1} \, \exp(-t) \,, \tag{3}$$

calculate by numerical integration, the following function values (you may use a computer algebra system but you must include a printout)

$$\Gamma(0.1) = ? \,, \qquad \Gamma(0.7) = ? \,, \qquad \Gamma(2.3) = ? \,. \tag{4}$$

Argue why the integral representation (3) cannot be used for, say, the case $z = -0.3$. Give a complex contour representation valid for the case $z < 0$, $z \neq -1, -2, -3, \ldots$. Calculate by the method of choice (you may use a computer algebra system but you must include a printout)

$$\Gamma(-0.1) = ? \,, \qquad \Gamma(-0.7) = ? \,, \qquad \Gamma(-2.3) = ? \,. \tag{5}$$

Finally, for 20 extra points, try to verify the result you obtain for $\Gamma(-0.7)$ by explicitly and numerically evaluating the complex contour integral on which the contour integral representation is based. Please note that this is the creative, and difficult, part of the exercise.

Task 4 (40 points)
Let $\rho = \rho(\vec{r}, t)$ by a time-dependent charge density, and let $\vec{J} = \vec{J}(\vec{r}, t)$ be a current density (unit is Ampere per square meter). Find the integral form of the differential charge conservation condition

$$\vec{\nabla} \cdot \vec{J}(\vec{r}, t) = \frac{\partial}{\partial t} \rho(\vec{r}, t) \,, \tag{6}$$

and argue why the integral form of this equation is tantamount to saying that total charge is conserved. *Hint:* Think about how much charge leaves the reference volume per unit time. Perhaps, think of the current as an analogue of flowing water. Use complete English sentences, and show your work!

The tasks are due Tuesday, 05-OCT-2021. Have fun doing the problems!