Task 1 (60 points)
(a.) Solve the Poisson equation (in three dimensions) with the help of the Green function,

$$\vec{\nabla}^2 \, \Phi(\vec{r}) = -\frac{1}{\epsilon_0} \, \rho(\vec{r}) \, , \qquad \rho(\vec{r}) = q_1 \, \delta^{(3)}(\vec{r} - \vec{r}_1) + q_2 \, \delta^{(3)}(\vec{r} - \vec{r}_2) \, . \tag{1}$$

Symbols are defined as in the lecture. Two point charges are places at $\vec{r}_1$ and $\vec{r}_2$. Show your work and all intermediate steps! **(b.)** Calculate the electric field

$$\vec{E}(\vec{r}) = -\vec{\nabla}\Phi(\vec{r}) \, . \tag{2}$$

(Of course, you are supposed to find an analytic expression involving the parameters q_1, q_2, $\vec{r}$, $\vec{r}_1$ and $\vec{r}_2$. Carefully distiguish independent arguments of functions, integration variables, and parameters!) **(c.)** Assume $q_1 = 0.04\,\mathrm{C}$ and $q_2 = 0.16\,\mathrm{C}$, $\vec{r}_1 = \vec{0}$, and $\vec{r}_2 = (2.5\,\mathrm{m})\,\hat{\mathrm{e}}_x$. Calculate the quantities

$$\Phi(\vec{r}_a) = ? \, , \qquad \vec{E}(\vec{r}_a) = ? \, , \qquad \vec{r}_a = (0.5\,\mathrm{m})\,\hat{\mathrm{e}}_x + (1.5\,\mathrm{m})\,\hat{\mathrm{e}}_y + (0.5\,\mathrm{m})\,\hat{\mathrm{e}}_z \, , \tag{3}$$

and

$$\Phi(\vec{r}_b) = ? \, , \qquad \vec{E}(\vec{r}_b) = ? \, , \qquad \vec{r}_b = (-0.5\,\mathrm{m})\,\hat{\mathrm{e}}_x + (-1.5\,\mathrm{m})\,\hat{\mathrm{e}}_y + (0.5\,\mathrm{m})\,\hat{\mathrm{e}}_z \, . \tag{4}$$

Write a computer program (in your favorite language!) and give numerical results for all three vector components of the electric fields at $\vec{r}_a$ and $\vec{r}_b$. Finally, calculate using the same computer program

$$\Phi_{\mathrm{diff}} = \Phi(\vec{r}_b) - \Phi(\vec{r}_a) \, . \tag{5}$$

Express your results in SI mksA units, preferably, to an accuracy of at least 4 decimals! The vacuum permittivity is $\epsilon_0 = 8.8542 \times 10^{-12}\mathrm{C\,V^{-1}\,m^{-1}}$, so that $1/(4\pi\epsilon_0) = 8.9875 \times 10^9\mathrm{N\,m^2/C^2}$.

Task 2 (60 points)
Write an essay to develop the concepts of a self-energy of an electrostatic field of a charge distribution, and the interaction energy of an electrostatic field of two charge distributions.

Show, by an explicit calculation, the formula

$$W = 2W_0 + W_{\mathrm{int}} = 2 \times \frac{q^2}{8\pi\epsilon_0 a} - \frac{q^2}{4\pi\epsilon_0 R} > 0 = \frac{q^2}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{R} \right) \, , \tag{6}$$

for the total energy stored in the electrostatic field of a configuration consisting of two uniformly charged spheres, each of radius a, of charges $+q$ and $-q$, a distance R apart. You may use lecture notes. SHOW WORK!!!!!

Also, calculate the total field energy (sum of self energies and interaction energies) of the electrostatic field of three (!) uniformly charged spheres, each of charge $-q$, at positions $\vec{x}_1$, $\vec{x}_2$ and $\vec{x}_3$.

The tasks are due Thursday, 21-OCT-2021. Have fun doing the problems!